(Continuation on §7.5)

Note Title 5/18/2006

If $A$ is real and an eigenvalue is complex, then the eigenvectors come in conjugate pairs.

$$\overline{A\vec{u}} = \overline{\lambda\vec{u}}, \qquad \overline{A\vec{u}} = \overline{A}\cdot\overline{\vec{u}} = \overline{\lambda}\cdot\overline{\vec{u}}.$$

$$\Longleftrightarrow A\overline{\vec{u}} = \overline{\lambda}\cdot\overline{\vec{u}}$$

5) Foundamental Thm of alg.

Any polyn can be factored as product of terms $(\lambda-\lambda_i)$, i.e.,

$$p(x) = a_n\lambda^n + a_{n-1}\lambda^{n-1} + \cdots + a_1\lambda + a_0$$
$$= a(\lambda-\lambda_1)\circ(\lambda-\lambda_2)\circ\cdots\circ(\lambda-\lambda_n)$$

eg $f(\lambda) = \lambda^2 - 2\lambda + 5$, $\quad \lambda = \dfrac{-(-2)\pm\sqrt{(-2)^2-4(1)5}}{2}$

$= (\lambda-1-2i)(\lambda-1+2i)$ $\quad = \dfrac{2\pm\sqrt{-16}}{2}$

$= 1\pm 2i$

5) $\boxed{\det(A) = \prod_{i=1}^{n}\lambda_i} \equiv \lambda_1\lambda_2\lambda_3\cdots\lambda_n$

Since $f_A(\lambda) = \prod_{i=1}^{n}(\lambda_i - \lambda)$ $\quad \therefore f_A(0) = \det(A - 0I) = \det(A)$

$= \prod_{i=1}^{n}\lambda_i$

eg #27 Facts: $\det(A) = \lambda_1\lambda_2\lambda_3 = \lambda_1^2\lambda_3 = 3$

$tr(A) = \lambda_1+\lambda_2+\lambda_3 = 2\lambda_1+\lambda_3 = 1$

could $(\lambda^2+1)^2$

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