Review 2

Math 325

Spring, 2006

Part I. All HW problems of covered sections of Chapters 3,5,6,7. Note again that Chapter 3 is included in the final. Review problems of Chapter 3 on Review 1.

Part II. Ture or False (Some of these problems require calculations)

1. The equation $(AB)^T = A^T B^T$ for all $n \times n$ matrices A and B.
2. All nonzero symmetric matrices are invertible.
3. If $\vec{u}$ is a unit vector in R^n, and $L = span(\vec{u})$, then $proj_L(\vec{x}) = (\vec{x} \cdot \vec{u})\vec{x}$ for all $\vec{x}$ in R^n.
4. If A and B are symmetric $n \times n$ matrices, then $ABBA$ must be symmetric as well.
5. There exists a subspace V of R^5 such that $dim(V) = dim(V^\perp)$, where $V^\perp$ is the orthogonal complement of V.
6. If $\vec{x}$ and $\vec{y}$ are two vectors in R^n, then the equation $\|\vec{x} + \vec{y}\|^2 = \|\vec{x}\|^2 + \|\vec{y}\|^2$ must hold.
7. If A is any matrix with $Ker(A) = \{\vec{0}\}$, then the matrix AA^T represents the orthogonal projection onto the range of A.
8. If V is a subspace of R^n and $\vec{x}$ is a vector in R^n, then vector $proj_V\vec{x}$ must be orthogonal to vector $\vec{x} - proj_V\vec{x}$.
9. The formula $ker(A) = ker(A^T A)$ holds for all matrices A.
10. If the entries of two vectors $\vec{v}$ and $\vec{w}$ in R^n are all positive, then $\vec{v}$ and $\vec{w}$ must enclose an acute angle.
11. The formula $ker(B^T) = im(B^T)$ holds for all matrices B.
12. The eigenvalues of any triangular matrix are its diagonal entries.
13. The algebraic multiplicity of an eigenvalue cannot exceed its geometric multiplicity.
14. If an $n \times n$ matrix A is diagonalizable (over R), then there must be a basis of R^n consisting of eigenvectors of A.
15. If the standard vectors $\vec{e}_1, \cdots, \vec{e}_n$ are eignevectors of an $n \times n$ matrix A, the A must be diagonal.
16. If $\vec{v}$ is an eigenvector of A, the $\vec{v}$ must be an eigenvector of A^3 as well.
17. If 0 is n eigenvalue of a matrix A, the $det(A) = 0$.
18. If 1 is the only eigenvalue of an $n \times n$ matrix A, then A must be I_n.
19. If A and B are $n \times n$ matrices, if α is an eigenvalue of A, and if β is an eigenvalue of B, then $\alpha\beta$ must be an eigenvalue of AB.
20. If 3 is an eigenvalue of A, then 9 must be an eigenvalue of A^2.
21. If two $n \times n$ matrices A and B are diagonalizable, the AB must be diagonalizable as well.
22. If an invertible matrix A is diagonalizable, then A^{-1} must be diagonalizable as well.
23. If vector $\vec{v}$ is an eigenvector of both A and B, then $vecv$ must be an eigenvector of $A + B$.
24. If an $n \times n$ matrix A is diagonalizable, then A must have n distict eigenvalues.
25. If a matrix is diagonalizable, then the algebraic multiplicity of each eigenvalue λ must equal the geometric multiplicity of λ.
26. If an $n \times n$ matrix A is diagonalizable (over R), then every vector $\vec{v}$ in R^n can be expressed as a sum of eigenvectors of A.
27. If A is a 2×2 matrix with eigenvalues 3 and 4, and if $\vec{u}$ is a unit eigenvector of A, then the length of the vector $A\vec{u}$ cannot exceed 4.