Review 2

Math 325

Spring, 2006

Part I. All HW problems of covered sections of Chapters 3,5,6,7. Note again that Chapter 3 is included in the final. Review problems of Chapter 3 on Review 1.

Part II. Ture or False (Some of these problems require calculations)

F 1. The equation $(AB)^T = A^T B^T$ for all $n \times n$ matrices A and B.

F 2. All nonzero symmetric matrices are invertible. $A=\begin{pmatrix}1&1\\1&1\end{pmatrix}$

F 3. If $\vec{u}$ is a unit vector in R^n, and $L = span(\vec{u})$, then $proj_L(\vec{x}) = (\vec{x}\cdot\vec{u})\cancel{\vec{x}}\,\vec{u}$ for all $\vec{x}$ in R^n.

T 4. If A and B are symmetric $n \times n$ matrices, then $ABBA$ must be symmetric as well. $(ABBA)^T = A^T B^T B^T A^T = ABBA$

F 5. There exists a subspace V of R^5 such that $dim(V) = dim(V^\perp)$, where $V^\perp$ is the orthogonal complement of V. $dim(V)+dim(V^\perp)=5$

F 6. If $\vec{x}$ and $\vec{y}$ are two vectors in R^n, then the equation $\|\vec{x}+\vec{y}\|^2 = \|\vec{x}\|^2 + \|\vec{y}\|^2$ must hold.

F 7. If A is any matrix with $Ker(A) = \{\vec{0}\}$, then the matrix AA^T represents the orthogonal projection onto the range of A. Fact 5.3.10 / Fact 5.4.7, true if A has orthonormal col. vectors

T 8. If V is a subspace of R^n and $\vec{x}$ is a vector in R^n, then vector $proj_V\vec{x}$ must be orthogonal to vector $\vec{x} - proj_V\vec{x}$. (Defn of orthog. proj)

T 9. The formula $ker(A) = ker(A^T A)$ holds for all matrices A. (Fact 5.4.2a)

T 10. If the entries of two vectors $\vec{v}$ and $\vec{w}$ in R^n are all positive, then $\vec{v}$ and $\vec{w}$ must enclose an acute angle. $\cos\theta = \dfrac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\,\|\vec{w}\|} > 0$

T 11. The formula $(ker(B))^\perp = im(B^T)$ holds for all matrices B. $(Ker B)^\perp = Im(B^T)$, Fact 5.4.1, $A=B^T$

T 12. The eigenvalues of any triangular matrix are its diagonal entries.

F 13. The algebraic multiplicity of an eigenvalue cannot exceed its geometric multiplicity.

T 14. If an $n \times n$ matrix A is diagonalizable (over R), then there must be a basis of R^n consisting of eigenvectors of A.

$A = AI_n = A[\vec{e}_1\,\vec{e}_2\cdots\vec{e}_n] = [A\vec{e}_1\,A\vec{e}_2\cdots A\vec{e}_n] = [\lambda_1\vec{e}_1\,\lambda_2\vec{e}_2\cdots\lambda_n\vec{e}_n]$

T 15. If the standard vectors $\vec{e}_1,\cdots,\vec{e}_n$ are eignevectors of an $n \times n$ matrix A, the A must be diagonal. $\left(=\begin{bmatrix}\lambda_1&&\\&\lambda_2&\\&&\ddots&\\&&&\lambda_n\end{bmatrix}\right)$

T 16. If $\vec{v}$ is an eigenvector of A, the $\vec{v}$ must be an eigenvector of A^3 as well. $A^3\vec{u} = A(A(A\vec{u})) = A(A\lambda\vec{u}) = \lambda^3\vec{u}$

T 17. If 0 is n eigenvalue of a matrix A, the $det(A) = 0$.

T 18. If 1 is the only eigenvalue of an $n \times n$ matrix A, then A must be I_n. $A=\begin{bmatrix}1&1\\0&1\end{bmatrix}$

F 19. If A and B are $n \times n$ matrices, if α is an eigenvalue of A, and if β is an eigenvalue of B, then $\alpha\beta$ must be an eigenvalue of AB. $A=\begin{bmatrix}2&0\\0&3\end{bmatrix}$, $B=\begin{bmatrix}4&0\\0&5\end{bmatrix}$, let $\alpha=2$, $\beta=5$, $AB=\begin{bmatrix}8&\\&15\end{bmatrix}$

T 20. If 3 is an eigenvalue of A, then 9 must be an eigenvalue of A^2.

F 21. If two $n \times n$ matrices A and B are diagonalizable, the AB must be diagonalizable as well. $A=\begin{bmatrix}1&1\\0&0\end{bmatrix}$ $B=\begin{bmatrix}0&1\\0&1\end{bmatrix}$ $AB=\begin{bmatrix}0&2\\0&0\end{bmatrix}$

T 22. If an invertible matrix A is diagonalizable, then A^{-1} must be diagonalizable as well. $S^{-1}AS=D \Rightarrow S^{-1}A^{-1}S=D^{-1}$

T 23. If vector $\vec{v}$ is an eigenvector of both A and B, then ~~vec~~ $\vec{v}$ must be an eigenvector of $A+B$. $(A+B)\vec{u} = A\vec{u}+B\vec{u} = \lambda_1\vec{u}+\lambda_2\vec{u} = (\lambda_1+\lambda_2)\vec{u}$

F 24. If an $n \times n$ matrix A is diagonalizable, then A must have n distict eigenvalues.

T 25. If a matrix is diagonalizable, then the algebraic multiplicity of each eigenvalue λ must equal the geometric multiplicity of λ.

T 26. If an $n \times n$ matrix A is diagonalizable (over R), then every vector $\vec{v}$ in R^n can be expressed as a sum of eigenvectors of A. eigenbasis

T 27. If A is a 2×2 matrix with eigenvalues 3 and 4, and if $\vec{u}$ is a unit eigenvector of A, then the length of the vector $A\vec{u}$ cannot exceed 4.

$A\vec{u} = \lambda\vec{u}$ $\quad \|A\vec{u}\| = |\lambda|\|\vec{u}\| = |\lambda|$